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Energy Scattering for the Generalized Davey-Stewartson Equations

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Abstract Considering the generalized Davey-Stewartson equation $i\dot{u} - \Delta u + \lambda|u|^p u + \mu E(|u|^q)|u|^{q-2}u = 0$, where $\lambda > 0$, $\mu \geqslant 0$, $E = \mathcal{F}^{-1}(\xi_1^2/|\xi|^2)\mathcal{F}$, we obtain the existence of scattering operator in $\Sigma(\mathbb{R}^n) := \{u \in H^1(\mathbb{R}^n) : |x|u \in L^2(\mathbb{R}^n)\}$.

Keywords Generalized Davey-Stewartson equation, pseudo conformally invariant conservation law, scattering operator
2000 MR Subject Classification 35P25, 35Q55

1 Introduction

In the present paper, we study the existence of the scattering operator for the generalized Davey-Stewartson equation

$$i\dot{u} - \Delta u + \lambda|u|^p u + \mu E(|u|^q)|u|^{q-2}u = 0, \tag{1.1}$$

where $\lambda > 0$, $\mu \geqslant 0$, $u = u(t,x) : \mathbb{R}^{1+n} \to \mathbb{C}$, $\dot{u} = \partial u/\partial t$, Δ is the Laplace operator on $\mathbb{R}^n$, $E = \mathcal{F}^{-1}(\xi_1^2/|\xi|^2)\mathcal{F}$, $\mathcal{F}$ denotes the Fourier and $\mathcal{F}^{-1}$ is its inverse transformation. Eq.(1.1) is a generalization of the following Davey-Stewartson system:

$$\begin{cases} iu_t + \delta u_{x_1x_1} + u_{x_2x_2} = \chi|u|^2u + bu\phi_{x_1}, \\ \phi_{x_1x_1} + m\phi_{x_2x_2} = (|u|^2)_{x_1}. \end{cases} \tag{1.2}$$

In the theory of surface waves of water waves, the 2D generalization of the usual cubic 1D Schrödinger equation turns out to be the Davey-Stewartson equation (cf. [2, 3]). The parameters δ and m are real, both χ and b are complex. A large amount of work has been devoted to the study of the Cauchy problem of equation (1.2) (e.g. [3, 5, 6, 8, 9, 10, 11, 13]). Ghidaglia and Saut[3], Guo and Wang[5] discussed the well-posedness of (1.2) and (1.1) ($q = 2$) in the energy space H^1, Wang and Guo[13] showed the existence of scattering operators in a "band" of H^s, where the Cauchy data should be suitably small in the critical spaces. In this paper, we shall use a different way considering the scattering operator of (1.1) for the large initial data. First, we will derive the pseudo conformally invariant conservation law of (1.1), which is similar to the nonlinear Schrödinger equation (cf. [4]). But some of the methods in [4] can not work on this kind of nonlinearity $E(|u|^q)|u|^{q-2}u$, we need to use some special properties of the operator E in the theory of Fourier analysis, and some techniques in [12]. By using the pseudo conformally invariant conservation law of (1.1) and applying the time-space $L^p - L^{p'}$ estimate method, we

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shall establish a decaying estimate and uniform boundedness for the solutions of (1.1), where we assume that the initial value

$$u(0,x) = u_0(x) \in \Sigma(\mathbb{R}^n) := \{u \in H^1(\mathbb{R}^n) : |x|u \in L^2(\mathbb{R}^n)\}$$

with the norm

$$\|u\|_\Sigma = \|u\|_{H^1} + \big\||x|u\big\|_{L^2}. \tag{1.3}$$

Whence, we can obtain that the scattering operator $S : \Sigma(\mathbb{R}^n) \to \Sigma(\mathbb{R}^n)$.

Now we state the main result of this paper.

Theorem 1.1. *Let $n \in \mathbb{N}$, p, $2q-2 \in [\frac{4}{n}, \frac{4}{n-2})$, ($\frac{4}{n-2} := \infty$ if $n = 1,2$), $S(t) = e^{-it\Delta}$, $\frac{2}{\gamma(r)} = n(\frac{1}{2} - \frac{1}{r})$, $\alpha(n) = \begin{cases} \frac{2n}{n-2}, & n \geqslant 3, \\ \infty & n = 1,2, \end{cases}$ $f(u) = \lambda |u|^p u + \mu E(|u|^q)|u|^{q-2}u$. If $u^- \in \Sigma$, then there exists a unique solution $u(t)$ of the integral equation*

$$u(t) = S(t)u^- + i\int_{-\infty}^t S(t-\tau) f\big(u(\tau)\big)d\tau \tag{1.4}$$

such that $u \in \mathcal{C}(\mathbb{R}, \Sigma) \cap L^{\gamma(r)}(\mathbb{R}, L^r)$, $\big(2 \leqslant r < \alpha(n)\big)$

$$\lim_{t\to-\infty} \|S(-t)u(t) - u^-\|_\Sigma = 0.$$

Moreover, there exists a unique $u^+ \in \Sigma$ such that

$$\lim_{t\to+\infty} \|S(-t)u(t) - u^+\|_\Sigma = 0.$$

In addition,

$$u(t) = S(t)u^+ - i\int_t^\infty S(t-\tau) f\big(u(\tau)\big)d\tau.$$

Thus the scattering operator $S : u^- \to u^+$ is a well-defined homeomorphism from Σ to itself.

Let us now describe the content of this paper. The first section is introduction. In the second section we shall derive three conservation identities including the L^2, the energy conservation law and the pseudo conformally invariant conservation law. Finally, we prove the main theorem in section 3. As a byproduct, the global existence and uniqueness for initial data in Σ are also obtained.

2 Derivation of the Conservation Laws

Proposition 2.1. *Let $n \in \mathbb{N}$, u be a solution of (1.1) with the initial value $u_0(x) \in \Sigma$. Then, we have the following conservation laws for all $t \in \mathbb{R}$:*

(i) *L^2-norm law:*

$$\|u(t)\|_2 = \|u_0\|_2;$$

(ii) *Energy conservation law:*

$$\begin{aligned}&\frac{1}{2}\|\nabla u(t)\|_2^2 + \frac{\lambda}{p+2}\|u(t)\|_{p+2}^{p+2} + \frac{\mu}{2q}\big\|E^{\frac{1}{2}}(|u(t)|^q)\big\|_2^2 \\ =&\frac{1}{2}\|\nabla u_0\|_2^2 + \frac{\lambda}{p+2}\|u_0\|_{p+2}^{p+2} + \frac{\mu}{2q}\big\|E^{\frac{1}{2}}(|u_0|^q)\big\|_2^2;\end{aligned}$$

where $E^{\frac{1}{2}} = \mathcal{F}^{-1}(|\xi_1|/|\xi|)\mathcal{F}$.

(iii) *Pseudo conformally invariant conservation law (the pcic law for short, cf. [4]):*

$$\|xu-2it\nabla u\|_2^2+\frac{8\lambda}{p+2}t^2\|u\|_{p+2}^{p+2}+\frac{4\mu}{q}t^2\big\|E^{\frac12}(|u|^q)\big\|_2^2+\frac{4\lambda}{p+2}(np-4)\int_0^t\tau\|u(\tau)\|_{p+2}^{p+2}d\tau$$
$$+\frac{4\mu}{q}(nq-n-2)\int_0^t\tau\big\|E^{\frac12}(|u(\tau)|^q)\big\|_2^2d\tau=\|xu_0\|_2^2.$$

Proof. Let $eq(u)=iu_t-\Delta u+\lambda|u|^pu+\mu E(|u|^q)|u|^{q-2}u$.

(i) It is known that it holds by virtue of the properties of the operator E. We omit its proof.

(ii) We consider

$$\mathrm{Re}\big(eq(u),u_t\big)=0,$$

where $(\cdot,\cdot)$ denotes L^2-inner product.

Noticing that

$$D_t\|u\|_{p+2}^{p+2}=(p+2)\mathrm{Re}(|u|^pu,u_t)$$

and

$$D_t\big\|E^{\frac12}(|u|^q)\big\|_2^2=2q\mathrm{Re}\big(E(|u|^q)|u|^{q-2}u,u_t\big),$$

we have

$$D_t\Big[\frac12\|\nabla u\|_2^2+\frac{\lambda}{p+2}\|u\|_{p+2}^{p+2}+\frac{\mu}{2q}\big\|E^{\frac12}(|u|^q)\big\|_2^2\Big]=0. \tag{2.1}$$

Integrating (2.1) over $[0,t]$, we obtain the desired identity.

(iii) We consider

$$\mathrm{Re}\big(eq(u)\bar u\big)=0.$$

Noticing that

$$\nabla(\nabla u\bar u)=\Delta u\bar u+|\nabla u|^2,$$

we have

$$-\mathrm{Im}u_t\bar u-\mathrm{Re}\nabla(\nabla u\bar u)+|\nabla u|^2+\lambda|u|^{p+2}+\mu E(|u|^q)|u|^q=0. \tag{2.2}$$

Next, we easily see that

$$\mathrm{Re}\big(eq(u)\bar u_rr\big)=0.$$

Noticing that

$$\mathrm{Re}\Delta u\bar u_rr=\nabla(\mathrm{Re}\nabla u\bar u_rr)-\nabla\Big(\frac x2|\nabla u|^2\Big)+\frac{n-2}{2}|\nabla u|^2,$$
$$\nabla(x|u|^{p+2})=n|u|^{p+2}+(p+2)\mathrm{Re}|u|^pu\bar u_rr,$$

we have

$$-\mathrm{Im}u_t\bar u_rr-\nabla\Big[\mathrm{Re}(\nabla u\bar u_rr)-\frac x2|\nabla u|^2\Big]-\frac{n-2}{2}|\nabla u|^2$$
$$+\frac{\lambda}{p+2}\big[\nabla(x|u|^{p+2})-n|u|^{p+2}\big]+\mu\mathrm{Re}E(|u|^q)|u|^{q-2}u\bar u_rr=0. \tag{2.3}$$

Consider

$$\partial_t(u\bar u_rr)+\nabla(xu_t\bar u)=2u_t\bar u_rr+2\mathrm{Re}xu\nabla\bar u_t+nu_t\bar u,$$

its imaginary part yields

$$\mathrm{Im}\partial_t(u\bar u_rr)+\mathrm{Im}\nabla(xu_t\bar u)=n\mathrm{Im}(u_t\bar u)+2\mathrm{Im}(u_t\bar u_rr). \tag{2.4}$$

From (2.2)–(2.4), we have

$$
\begin{aligned}
&-\operatorname{Im}\partial_t(u\bar{u}_r r)-\operatorname{Im}\nabla(xu_t\bar{u})-n\operatorname{Re}\nabla(\nabla u\bar{u})+2|\nabla u|^2+n\lambda|u|^{p+2}+n\mu E(|u|^q)|u|^q\\
&-2\nabla\Big[\operatorname{Re}(\nabla u\bar{u}_r r)-\frac{x}{2}|\nabla u|^2\Big]+\frac{2\lambda}{p+2}\big[\nabla(x|u|^{p+2})-n|u|^{p+2}\big]\\
&+2\mu\operatorname{Re}E(|u|^q)|u|^{q-2}u\bar{u}_r r=0. \qquad (2.5)
\end{aligned}
$$

Denoting (2.5) by

$$\frac{\partial X}{\partial t}+\nabla\cdot Y+Z+S=0, \qquad (2.6)$$

where

$$X=-\operatorname{Im}(u\bar{u}_r r), \qquad (2.7)$$

$$Y=-n\operatorname{Re}\nabla u\bar{u}-2\operatorname{Re}(\nabla u\bar{u}_r r)+x|\nabla u|^2+\frac{2\lambda}{p+2}x|u|^{p+2}-\operatorname{Im}xu_t\bar{u}, \qquad (2.8)$$

$$Z=2|\nabla u|^2+\frac{np\lambda}{p+2}|u|^{p+2}+n\mu E(|u|^q)|u|^q, \qquad (2.9)$$

$$S=2\mu\operatorname{Re}E(|u|^q)|u|^{q-2}u\bar{u}_r r, \qquad (2.10)$$

we also have

$$\partial_t|xu-2it\nabla u|^2=\partial_t|ur|^2-4\operatorname{Im}xu\nabla\bar{u}-4t(\nabla Y+Z+S)+4\partial_t(t^2|\nabla u|^2). \qquad (2.11)$$

Since

$$\operatorname{Im}\nabla(\nabla u\bar{u}r^2)=\operatorname{Im}\Delta u\bar{u}r^2+2\operatorname{Im}\nabla u\bar{u}x$$

and

$$\frac{1}{2}\partial_t|ur|^2=\operatorname{Im}\Delta u\bar{u}r^2,$$

we have

$$\partial_t|xu-2it\nabla u|^2=2\operatorname{Im}\nabla(\nabla u\bar{u}r^2)-4t(\nabla Y+Z+S)+8t|\nabla u|^2+4t^2\partial_t|\nabla u|^2. \qquad (2.12)$$

Integrating (2.12) over $\mathbb{R}^n$ and in view of (2.1), we obtain that

$$
\begin{aligned}
&D_t\Big[\|xu-2it\nabla u\|_2^2+\frac{8\lambda}{p+2}t^2\|u\|_{p+2}^{p+2}+\frac{4\mu}{q}t^2\big\|E^{\frac{1}{2}}(|u|^q)\big\|_2^2\Big]+\frac{4\lambda}{p+2}(np-4)t\|u\|_{p+2}^{p+2}\\
&+\frac{4\mu}{q}(nq-2)t\big\|E^{\frac{1}{2}}(|u|^q)\big\|_2^2+\int_{\mathbb{R}^n}\operatorname{Re}E(|u|^q)|u|^{q-2}u\bar{u}_r r\,dx=0. \qquad (2.13)
\end{aligned}
$$

Now, we estimate

$$
\begin{aligned}
\int_{\mathbb{R}^n}\operatorname{Re}E(|u|^q)|u|^{q-2}u\bar{u}_r r\,dx&=\int_{\mathbb{R}^n}E(|u|^q)|u|^{q-2}x\operatorname{Re}u\nabla\bar{u}\,dx=\frac{1}{q}\int_{\mathbb{R}^n}E(|u|^q)x\cdot\nabla(|u|^q)dx\\
&=\frac{1}{q}\int_{\mathbb{R}^n}\frac{\xi_1^2}{|\xi|^2}(\mathcal{F}|u|^q)\mathcal{F}\big(x\nabla(|u|^q)\big)d\xi\\
&=-\frac{n}{q}\big\|E^{\frac{1}{2}}(|u|^q)\big\|_2^2-\frac{1}{q}\int_{\mathbb{R}^n}\frac{\xi_1^2}{|\xi|^2}(\mathcal{F}|u|^q)\xi\cdot\nabla(\mathcal{F}|u|^q)d\xi.
\end{aligned}
$$

Let $v=\mathcal{F}|u|^q$, we have

$$4t\int S\,dx=-\frac{4n\mu t}{q}\big\|E^{\frac{1}{2}}(|u|^q)\big\|_2^2. \qquad (2.14)$$

Hence, we have from (2.13)

$$D_t\Big[\|xu-2it\nabla u\|_2^2+\frac{8\lambda}{p+2}t^2\|u\|_{p+2}^{p+2}+\frac{4\mu}{q}t^2\big\|E^{\frac12}(|u|^q)\big\|_2^2\Big]+\frac{4\lambda}{p+2}(np-4)t\|u\|_{p+2}^{p+2}$$
$$+\frac{4\mu}{q}(nq-n-2)t\big\|E^{\frac12}(|u|^q)\big\|_2^2=0. \tag{2.15}$$

Integrating (2.15) over $[0,t]$, we obtain the pcic law.

3 Proof of Theorem 1.1

Now we recall the lemma needed to prove this theorem.

Lemma 3.1. (cf. [12, Proposition 0.1] or [7, Theorem 10.1]) *Let* $2\leqslant r,\ q\leqslant \alpha(n)$ $(r,\ q\neq \infty$ *if* $n=2)$, $S(t)=e^{-it\Delta}$. *Then there exists a constant* $C>0$ *such that*

$$\|S(t)\varphi\|_{L^{\gamma(r)}(I;B^s_{r,2})}\leqslant C\|\varphi\|_{H^s}, \tag{3.1}$$

$$\Big\|\int_{\tau<t}S(t-\tau)fd\tau\Big\|_{L^{\gamma(r)}(I;B^s_{r,2})}\leqslant C\|f\|_{L^{\gamma(q)'}(I;B^s_{q',2})} \tag{3.2}$$

hold for all $\varphi\in H^s$, $f\in L^{\gamma(q)'}(I;B^s_{q',2})$ *and for all* $I\subset\mathbb{R}$, *where* $1/p+1/p'=1$.

Remark. Both (3.1) and (3.2) hold true if we replace $B^s_{r,2}$ by H^s_r when $s\in\mathbb{N}$ (see [1, 7]).

Proof of Theorem 1.1. It is clear that Proposition 2.1 holds if we replace u_0 by u^-.

Using Proposition 2.1, we can get

$$\|u\|_{p+2}\leqslant C,\qquad \|u\|_{H^1}\leqslant C.$$

Thus, we obtain by the Sobolev embedding and the interpolation inequality

$$\|u\|_r\leqslant c\|u\|_{H^{1-(p+2)/r}_{\frac{2r}{r-p}}}\leqslant C\|u\|_{H^1}^{1-(p+2)/r}\|u\|_{p+2}^{(p+2)/r}\leqslant C,\qquad p+2\leqslant r<\alpha(n)$$

and from the energy equality, we have for $2\leqslant r\leqslant p+2$,

$$\|u\|_r\leqslant C\|u\|_2^{1-\theta}\|u\|_{p+2}^{\theta}\leqslant C,$$

where $\theta=\frac{r-2}{p}\cdot\frac{p+2}{r}\in[0,1]$. Hence, we obtain

$$\|u\|_r\leqslant C,\qquad 2\leqslant r<\alpha(n). \tag{3.3}$$

On the other hand, by the Sobolev embedding and the pcic law, we have

$$\|u\|_r=\big\|e^{-\frac{|x|^2}{4it}}u\big\|_r\leqslant C\big\|\nabla(e^{-\frac{|x|^2}{4it}}u)\big\|_2^{2/\gamma(r)}\big\|e^{-\frac{|x|^2}{4it}}u\big\|_2^{1-2/\gamma(r)}$$
$$\leqslant C\Big\|e^{-\frac{|x|^2}{4it}}\Big(\frac{-2x}{4it}u+\nabla u\Big)\Big\|_2^{2/\gamma(r)}\|u\|_2^{1-2/\gamma(r)}\leqslant C|t|^{-2/\gamma(r)}. \tag{3.4}$$

Therefore, we obtain

$$\|u\|_r\leqslant C\min\{1,|t|^{-2/\gamma(r)}\},\qquad 2\leqslant r<\alpha(n) \tag{3.5}$$

and

$$\|u\|_{L^{\gamma(r)}(\mathbb{R};L^r)}\leqslant C,\qquad 2\leqslant r<\alpha(n). \tag{3.6}$$

Noticing that

$$xS(-t)u = S(-t)(xu - 2it\nabla u), \tag{3.7}$$

we have via the pcic law

$$\|xS(-t)u\|_{L^\infty(\mathbb{R};L^2)} \leqslant C.$$

So we have from the property of the operator $S(t)$ (cf. [4])

$$u \in \mathcal{C}(\mathbb{R}, \Sigma).$$

Thus, we proved $u \in \mathcal{C}(\mathbb{R}, \Sigma) \cap L^{\gamma(r)}(\mathbb{R}; L^r)$, $2 \leqslant r < \alpha(n)$.

Now we introduce the Galilei-type operator

$$J(t) = x - 2it\nabla. \tag{3.8}$$

By (3.7) and (3.4), we acquire

$$J(t) = S(t)xS(-t), \tag{3.9}$$

$$J(t)u(t) = -2ite^{\frac{|x|^2}{4it}}\nabla\big(e^{-\frac{|x|^2}{4it}}u(t)\big). \tag{3.10}$$

Let $w(t) = e^{-\frac{|x|^2}{4it}}u(t)$, we get

$$|w(t)| = |u(t)|, \qquad |J(t)u(t)| = 2|t|\,|\nabla w(t)|. \tag{3.11}$$

Owing to (1.4), we have

$$S(-t)u(t) = u^- + i\int_{-\infty}^t S(-\tau)f\big(u(\tau)\big)d\tau, \tag{3.12}$$

$$J(t)u(t) = S(t)xu^- + i\int_{-\infty}^t S(t-\tau)J(\tau)f\big(u(\tau)\big)d\tau. \tag{3.13}$$

By (3.4), we have for $T < 0$ and $r > 2 + 2/n$

$$\|u\|_{L^r(-\infty,T;L^r)} \leqslant C\Big(\int_{-\infty}^T \big(|t|^{-n(1/2-1/r)}\big)^r dt\Big)^{1/r} \leqslant C|T|^{-n(r-2-2/n)/2r}. \tag{3.14}$$

In view of (3.10), it is clear that

$$J(\tau)f\big(u(\tau)\big) = -2i\tau e^{\frac{|x|^2}{4i\tau}}\nabla\big(f\big(w(\tau)\big)\big) \tag{3.15}$$

and

$$\big|Jf\big(u(\tau)\big)\big| = 2|\tau|\,\big|\nabla f\big(w(\tau)\big)\big|. \tag{3.16}$$

Let $\rho = 2 + 4/n$, $1/\rho + 1/\rho' = 1$, we have

$$\big\|Jf\big(u(\tau)\big)\big\|_{\rho'} = 2|\tau|\,\big\|\nabla f\big(w(\tau)\big)\big\|_{\rho'}. \tag{3.17}$$

Since

$$\nabla f(w) = \lambda(\nabla|w|^p)w + \lambda|w|^p\nabla w + \mu E(\nabla|w|^q)|w|^{q-2}w + \mu E(|w|^q)\nabla(|w|^{q-2}w),$$

we obtain from the Hölder's inequality and the properties of the operator E, i.e. $E \in M_p$, $1 < p < \infty$ (cf. [1]), that

$$\|\nabla f(w)\|_{\rho'} \leqslant C\big(\|w\|_{m_1}^p + \|w\|_{m_2}^{2q-2}\big)\|\nabla w\|_\rho, \tag{3.18}$$

where $\frac{p}{m_1} = \frac{2q-2}{m_2} = \frac{1}{\rho'} - \frac{1}{\rho} = \frac{2}{n+2}$ (obviously, $m_1, m_2 \geqslant 2+4/n > 2+2/n$). Then, by (3.15), (3.16) and (3.11), we see that

$$\big\|Jf\big(u(\tau)\big)\big\|_{\rho'} \leqslant C\big(\|u\|_{m_1}^p + \|u\|_{m_2}^{2q-2}\big)\|Ju(\tau)\|_\rho. \tag{3.19}$$

So we have by the Hölder's inequality and (3.14)

$$\begin{aligned}\big\|Jf(u)\big\|_{L^{\rho'}(-\infty,T;L^{\rho'})} &\leqslant C\big(\|u\|^p_{L^{m_1}(-\infty,T;L^{m_1})} + \|u\|^{2q-2}_{L^{m_2}(-\infty,T;L^{m_2})}\big)\|Ju\|_{L^\rho(-\infty,T;L^\rho)}\\ &\leqslant C\beta(T)\|Ju\|_{L^\rho(-\infty,T;L^\rho)},\end{aligned} \tag{3.20}$$

where $\beta(T) := |T|^{-np(m_1-2-2/n)/2m_1} + |T|^{-n(2q-2)(m_2-2-2/n)/2m_2}$. Thus, from (3.13) and Lemma 3.1, we have

$$\begin{aligned}\|Ju\|_{L^\rho(-\infty,T;L^\rho)} &\leqslant C\|xu^-\|_2 + \big\|Jf\big(u(\tau)\big)\big\|_{L^{\rho'}(-\infty,T;L^{4/3})}\\ &\leqslant C\|xu^-\|_2 + C\beta(T)\|Ju\|_{L^\rho(-\infty,T;L^\rho)}.\end{aligned} \tag{3.21}$$

Taking $T_0 > 0$ sufficiently large so that

$$C\beta(T_0) \leqslant \frac{1}{2},$$

we have for $T < -T_0$

$$\|Ju\|_{L^\rho(-\infty,T;L^\rho)} \leqslant C\|u^-\|_\Sigma. \tag{3.22}$$

Due to (3.13), (3.20) and (3.22), we have for $T < -T_0$

$$\begin{aligned}&\big\|xS(-t)u(t) - xu^-\big\|_{L^\infty(-\infty,T;L^2)} = \big\|J(t)u(t) - S(t)xu^-\big\|_{L^\infty(-\infty,T;L^2)}\\ \leqslant &C\big\|Jf(u)\big\|_{L^{\rho'}(-\infty,T;L^{\rho'})} \leqslant C\beta(T) \to 0, \qquad T \to -\infty.\end{aligned}$$

Thus, we obtain

$$\big\|xS(-t)u(t) - xu^-\big\|_2 \to 0, \qquad t \to -\infty. \tag{3.23}$$

On the other hand, by (1.4), the Strichartz inequality and (3.14), we have for $T < -T_0$

$$\begin{aligned}\|u\|_{L^\rho(-\infty,T;H^1_\rho)} &\leqslant C\|u^-\|_{H^1} + C\|f(u)\|_{L^{\rho'}(-\infty,T;H^1_{\rho'})}\\ &\leqslant C\|u^-\|_{H^1} + C\big(\|u\|^p_{L^{m_1}(-\infty,T;L^{m_1})} + \|u\|^{2q-2}_{L^{m_2}(-\infty,T;L^{m_2})}\big)\|u\|_{L^\rho(-\infty,T;H^1_\rho)}\\ &\leqslant C\|u^-\|_{H^1} + \frac{1}{2}\|u\|_{L^\rho(-\infty,T;H^1_\rho)}.\end{aligned}$$

Thus,

$$\|u\|_{L^\rho(-\infty,T;H^1_\rho)} \leqslant C. \tag{3.24}$$

From (3.12), (3.14) and (3.24), we get

$$\big\|S(-t)u(t) - u^-\big\|_{L^\infty(-\infty,T;H^1)} \leqslant C\|f(u)\|_{L^{\rho'}(-\infty,T;H^1_{\rho'})} \leqslant C\beta(T) \to 0, \qquad T \to -\infty.$$

Thus, we have

$$\|S(-t)u(t) - u^-\|_{H^1} \to 0, \qquad t \to -\infty.$$

Hence, we obtain

$$\|S(-t)u(t) - u^-\|_\Sigma \to 0, \qquad t \to -\infty.$$

We now define

$$u^+(t) = u(t) + i\int_t^{+\infty} S(t-\tau)f\big(u(\tau)\big)d\tau$$

and let $u^+(0) = u^+$. Then we show in the same way as above that

$$\left\| S(-t)u(t) - u^+ \right\|_\Sigma \to 0, \qquad t \to +\infty.$$

The proof of the uniqueness of $u(t)$ and u^+ is standard, one can refer to [3, 4]. Thus, the existence of the scattering operator $S : u^- \to u^+$ is established.

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References

[1] Bergh, J., Löfström, J. Interpolation space. Springer-Verlag, Berlin, 1976
[2] Davey, A., Stewartson, K. On three dimensional packets of surface waves. *Proc. Roy. Soc.* (Ser. A), 338: 101–110 (1976)
[3] Ghidaglia, J.M., Saut, J.C. On the initial value problem for the Davey-Stewartson systems. *Nonlinearity*, 3: 475–506 (1990)
[4] Ginibre, J., Velo, G. On a class of nonlinear Schrödinger equations. *J. Funct. Anal.*, 32: 1–72. (1979)
[5] Guo, B., Wang, B.X. The Cauchy problem for Davey-Stewartson systems. *Comm. Pure Appl. Math.*, 52(12): 1477–1490 (1999)
[6] Hayashi, N., Saut, J.C. Global existence of small solutions to the Davey-Stewartson and the Ishimori systems. *Differential Integral Equations*, 8(7): 1657–1675 (1995)
[7] Keel, M., Tao, T. *Endpoint Strichartz estimate*. Amer. J. Math., 120: 955–980 (1998)
[8] Ohta, M. Instability of standing waves for the generalized Davey-Stewartson system. *Ann. Inst. H. Poincaré Phys. Théor*, 62(1): 69–80 (1995)
[9] Ohta, M. Stability of standing waves for the generalized Davey-Stewartson system. *J. Dynam. Differential Equations*, 6(2): 325–334 (1994)
[10] Ozawa, T. Exact blow-up solutions to the Cauchy problem for the Davey-Stewartson systems. *Proc. Roy. Soc. London* (Ser. A), 436(1897): 345–349 (1992)
[11] Sung, L.Y. Long-time decay of the solutions of the Davey-Stewartson II equations. *J. Nonlinear Sci.*, 5(5): 433–452 (1995)
[12] Wang, B.X., Large time behavior of solutions for critical and subcritical complex Ginzburg-Landau equations in H^1. *Science in China* (Series A), 32(7): 657–666 (2001)
[13] Wang, B.X., Guo, B.L. On the initial value problem and scattering of solutions for the generalized Davey-Stewartson systems. *Science in China* (Series A), 44(8): 994–1002 (2001)