Mathematical Models and Methods in Applied Sciences
Vol. 14, No. 10 (2004) 1481–1494

MODIFIED SCATTERING FOR BIPOLAR NONLINEAR SCHRÖDINGER–POISSON EQUATIONS

CHENGCHUN HAO

Academy of Mathematics and Systems Science, CAS,
Beijing 100080, P. R. China
Graduate School of the Chinese Academy of Sciences,
Beijing 100039, P. R. China
hcc@mail.amss.ac.cn

LING HSIAO

Academy of Mathematics and Systems Science, CAS,
Beijing 100080, P. R. China
hsiaol@mail.amss.ac.cn

HAILIANG LI

Department of Mathematics, Capital Normal University,
Beijing 100037, P. R. China

Received 3 December 2003
Revised 31 March 2004
Communicated by P. Degond

In this paper, we study the asymptotic behavior in time and the existence of the modified scattering operator of the globally defined smooth solutions to the Cauchy problem for the bipolar nonlinear Schrödinger–Poisson equations with small data in the space $\mathbb{R}^3$.

Keywords: Bipolar nonlinear Schrödinger–Poisson; Cauchy problem; global existence; modified scattering operator.

AMS Subject Classification: 35Q55, 82D10

1. Introduction

The nonlinear Schrödinger–Poisson system is used to simulate the transport of charged particles in semiconductor science and plasma physics.[22] In the present paper, we study the asymptotic behavior in time and the existence of the modified scattering operator of the solutions to the Cauchy problem for the bipolar nonlinear Schrödinger–Poisson equations of two carriers

$$i\dot{\psi}_j = -\frac{1}{2}\Delta\psi_j + (q_j V(\psi_1, \psi_2) + a_j^2|\psi_j|^p)\psi_j\,, \tag{1.1}$$

$$V = \frac{1}{4\pi|x|} * (|\psi_1|^2 - |\psi_2|^2)\,, \tag{1.2}$$

$$\psi_j(0,x) = \phi_j(x), \quad x \in \mathbb{R}^3\,, \tag{1.3}$$

where Δ denotes the Laplacian on $\mathbb{R}^3$ and $\dot{\psi}_j = \partial\psi_j/\partial t$. The wave function $\psi_j = \psi_j(t,x)$: $\mathbb{R}^{1+3} \to \mathbb{C}$, $j = 1,2$, describes the state of the particle in the position space under the action of the electrostatic potential $V = V(t,x)$ at every instant t. The charges of the particles described by the wave functions ψ_j are defined by $q_1 = 1$, $q_2 = -1$, respectively. We also assume that $4/3 < p < 4$ in the nonlinear self-interaction potential $a_j^2|\psi_j|^p$ where $a_j \in \mathbb{R}$.

A large amount of interesting work has been devoted to the mathematical analysis for the bipolar Schrödinger–Poisson system[13,17] or for the unipolar Schrödinger–Poisson system, the Hartree equation[3,4,8–10,19,21] and the Schrödinger equation.[2,5,7–12,16,23–25]

In the unipolar case, Castella[4] proved the global existence (of L^2-solution, in particular) and the asymptotic behavior of solutions in the function space L^2 for the mixed-state unipolar Schrödinger–Poisson systems without the defocusing nonlinearity. López and Soler[20] discussed the large-time behavior of the solutions to the unipolar Schrödinger–Poisson equations without nonlinearities, by using an appropriate scaling group and the equivalence between the Schrödinger formalism and the Wigner representation of quantum mechanics. They proved that, when time went to infinity, the limit of the rescaled self-consistent potential can be identified as the Coulomb potential. With the help of WKB-ansatz, Li and Lin[19] discussed the unipolar nonlinear Schrödinger–Poisson system with frictional damping subject to the rapidly oscillating (WKB) initial data and obtained the semiclassical convergence to the compressible Euler–Poisson equations for smooth solutions in a finite time interval. Based on the techniques of scattering theory, Hayashi and Naumkin[14] established the modified scattering theory for nonlinear Schrödinger equations and Hartree equations respectively.

Concerning bipolar equations, by applying the estimates of a modulated energy functional and the Wigner measure method, Jüngel and Wang[17] discussed the combined semiclassical and quasineutral limit of the solutions to the bipolar nonlinear Schrödinger–Poisson equations in the whole space where $a_1 = a_2$ provided that there exists a solution to the Cauchy problem. By using the pseudo-conformal conservation law of the bipolar nonlinear Schrödinger–Poisson system and applying the time-space $L^p - L^{p'}$ estimate method, Hao and Hsiao[13] have established the global existence and uniqueness and large-time behavior of the solution to the bipolar defocusing nonlinear Schrödinger–Poisson system with initial data in $\Sigma := \{\phi \in H^1(\mathbb{R}^3) : |x|\phi \in L^2(\mathbb{R}^3)\}$ and proved that the solution (ψ_1, ψ_2, V) satisfies

$$\psi_j \in \mathcal{C}(\mathbb{R};\, \Sigma(\mathbb{R}^3)) \cap L^\infty(\mathbb{R};\, H^1(\mathbb{R}^3)) \cap L^{\gamma(\rho)}_{\text{loc}}(\mathbb{R};\, H^1_\rho(\mathbb{R}^3)), \quad \text{for } j = 1,2,$$

where $\rho \in [2,6)$, $\frac{1}{\gamma(\rho)} = \frac{3}{2}(\frac{1}{2} - \frac{1}{\rho})$ and H^1_ρ is the usual Sobolev space. And in large time, the wave function and the potential decay, for any $\rho_1 \in (2,6), \rho_2 \in (\frac{3}{2},\infty), \rho_3 \in (3,\infty)$, as follows

$$\|\psi_j(t)\|_{\rho_1} \le C|t|^{-\frac{1}{\gamma(\rho_1)}}, \quad \|\nabla V(t)\|_{\rho_2} \le C|t|^{-(1-\frac{3}{2\rho_2})}, \quad \|V(t)\|_{\rho_3} \le C|t|^{-\frac{1}{2}(1-\frac{3}{\rho_3})},$$

where the norm $\|\cdot\|_\rho$ denotes $\|\cdot\|_{L^\rho}$, C is a positive constant depending only on p, $\|\phi_j\|_{H^1}$ and $\||x|\phi_j\|_2$. But there is no result of the existence of scattering operator since the decay estimates obtained are not enough to prove the existence of scattering operator.

It is well known that nontrivial solution of the Hartree equation, say,

$$i\dot{u} = -\frac{1}{2}\Delta u + \lambda|x|^{-1} * |u|^2 \tag{1.4}$$

does not approach any free solution as time tends to infinity[15] where $\lambda \in \mathbb{R}$. But we need some modification in the phase to approximate the solutions, which is described by the modified wave operators.[7] There are at least three kinds of approximation in the literature, namely

$$v_1^*(t) = S(t)e^{\frac{i}{2}V(\varphi)\ln t}\varphi, \tag{1.5}$$

$$v_2^*(t) = e^{\frac{i}{2}V(\varphi)\ln t}\varphi, \tag{1.6}$$

$$v_3^*(t) = e^{\frac{i}{2}V(\varphi)\ln t}S(t)\varphi, \tag{1.7}$$

which correspond to v_1, v_2, v_3 respectively in (2.8)–(2.10)[8] where $S(t) = e^{\frac{1}{2}i\Delta t}$ as defined below. The first one turns out to be the best to avoid the derivative loss which cause unpleasant mismatch of the topologies for asymptotic states.[24]

In the scattering theory of nonlinear Schrödinger equations, there have been a lot of important works.[2,5,11,12,16,23] Ginibre and Velo gave the general scattering theory.[11,12] Lin and Strauss[16] used a Morawetz inequality to obtain an *a priori* estimate. Bourgain introduced the localize Morawetz estimates and decomposition in low and high frequency.[2] Nakanishi[23] used methods which were based on the separation of the localized energy argument. And the method used by Tao *et al.*[5] was similar in spirit to the induction-on-energy strategy of Bourgain,[2] but the authors performed the induction analysis in both frequency space and physical space simultaneously, and replaced the Morawetz inequality by an interaction variant.

Although there are already many results on the large-time behavior for the solutions to the nonlinear Schrödinger–Poisson equations and the scattering theory for the nonlinear Schrödinger equation[6,15,25,26] or the Hartree equation,[6,8–10,14,15] we cannot find the scattering theory for the bipolar Schrödinger–Poisson system with the power nonlinearity from the previous known results. In the present paper, we will establish the scattering theory for the bipolar Schrödinger–Poisson system. We try to explain what the scattering operator will be in the bipolar case and to find the modification caused by the power nonlinearity. It turns out that we can roughly neglect the affection of the power contribution from the view of scattering analysis when the power is noncritical. The main difficulty is how to deal with the Coulomb potential which is critical for scattering analysis. With the help of the argument developed by Hayashi and Naumkin,[14] we can overcome this difficulty, provided the data are small.

We consider the Cauchy problem to the Schrödinger–Poisson equations (1.1)–(1.2) under the following condition on initial data

$$\phi_j \in H^{\gamma,0} \cap H^{0,\gamma}, \quad \text{with} \quad \gamma > 3/2,\ j = 1,2$$

and the norm $\sum_{j=1,2} \|\phi_j\|_{\gamma,0} + \|\phi_j\|_{0,\gamma}$ is sufficiently small, where the space $H^{\gamma,\nu}$ is the usual weighted Sobolev space defined by

$$H^{\gamma,\nu} := \{u \in L^2 : \|u\|_{\gamma,\nu} = \|(1+|x|^2)^{\nu/2}(1-\Delta)^{\gamma/2}u\|_2 < \infty\},\ \gamma,\nu \in \mathbb{R}. \tag{1.8}$$

For convenience, we first introduce some notations. $S(t)$ denotes the unitary group generated by $\frac{1}{2}i\Delta$ in $L^2(\mathbb{R}^3)$. $(D(t)u)(x) = (it)^{-3/2}u(x/t)$, $D(t)^{-1} = i^3 D(1/t)$, $M = M(t) = e^{\frac{i|x|^2}{2t}}$, $J = S(t)xS(-t)$ and $|J|^\beta = S(t)|x|^\beta S(-t)$, $\beta \in [0,\infty)$. $\bar{z}$ denotes the conjugate of the complex number z. $\mathcal{F}u$ or $\hat{u}$ ($\mathcal{F}^{-1}u$, respectively) denotes the Fourier (inverse, respectively) transform of u. In this paper, the constant C might be different from each other in the different position.

We now state our results on global existence and modified scattering in large time as follows.

Theorem 1.1. *We assume that* $\phi_j \in H^{\gamma,0} \cap H^{0,\gamma}$ *and* $\sum_{j=1,2}[\|\phi_j\|_{\gamma,0} + \|\phi_j\|_{0,\gamma}] =: \varepsilon_1 \le \varepsilon$, *where* ε *is sufficiently small and* $3/2 < \gamma \le 5/3$. *Then there exists a unique global solution* (ψ_1, ψ_2, V) *of* (1.1)–(1.2) *with the initial data* (1.3) *such that for* $j = 1,2$

$$\psi_j \in \mathcal{C}(\mathbb{R}; H^{\gamma,0} \cap H^{0,\gamma}), \quad \|\psi_j(t)\|_\infty \le C\varepsilon_1(1+|t|)^{-3/2},$$
$$\|V(t)\|_\infty \le C\min\{\varepsilon_1{}^2(1+|t|)^{-1}, \varepsilon_1|t|^{-\alpha+C(\varepsilon_1^2+\varepsilon_1^p)}\},\ t \in \mathbb{R},$$

where $C(\varepsilon_1^2 + \varepsilon_1^p) < C\varepsilon < \alpha < 1$, $4/3 < p < 4$ *and* C *is a finite number which is independent of* ε *and* ε_1.

Theorem 1.2. *Let* (ψ_1, ψ_2) *be the solution of* (1.1)–(1.2) *with* (1.3) *obtained in Theorem* 1.1. *Then for any* $\phi_j \in H^{\gamma,0} \cap H^{0,\gamma}, j = 1,2$, *there exist a unique pair of functions* $(\mathcal{W}_1, \mathcal{W}_2)^\pm$ *with* $\mathcal{W}_j^\pm \in L^\infty$, $j = 1,2$, *and a real-valued function* $\Lambda^\pm \in L^\infty$ *for* $t \to \pm\infty$, *respectively, such that*

$$\left\| \mathcal{F}(S(-t)\psi_j(t)) e^{-iq_j \int_{\wedge(t)}^{\vee(t)} V(\hat{\psi}_1,\hat{\psi}_2)\frac{d\tau}{|\tau|}} - \mathcal{W}_j^\pm \right\|_\infty \le C\varepsilon_1 |t|^{-\alpha + C(\varepsilon_1^2+\varepsilon_1^p)}, \tag{1.9}$$

and

$$\left\| \int_{\wedge(t)}^{\vee(t)} V(\hat{\psi}_1,\hat{\psi}_2)|\tau|^{-1}\,d\tau - V(\mathcal{W}_1^\pm, \mathcal{W}_2^\pm)\ln|t| - \Lambda^\pm \right\|_\infty \le C\varepsilon_1 |t|^{\theta(-\alpha+C(\varepsilon_1^2+\varepsilon_1^p))}, \tag{1.10}$$

where

$$\wedge(t) = \begin{cases} 1, & t \ge 1 \\ t, & t \le -1 \end{cases},\ \vee(t) = \begin{cases} t, & t \ge 1 \\ -1, & t \le -1 \end{cases},\ 0 < \theta < 2/3, C\varepsilon < \alpha < 1$$

and $\gamma > 3/2 + 2\alpha$*. We recall that* ε_1 *is defined in Theorem* 1.1*. Furthermore, we have the estimate for* $t \to \pm\infty$ *that*

$$\left\| \mathcal{F}(S(-t)\psi_j) - \mathcal{W}_j^{\pm} e^{iq_j(V(\mathcal{W}_1^{\pm},\mathcal{W}_2^{\pm})\ln|t| + \Lambda^{\pm})} \right\|_{\infty} \le C\varepsilon_1 |t|^{\theta(-\alpha + C(\varepsilon_1^2 + \varepsilon_1^p))} . \qquad (1.11)$$

2. Proofs of the Main Theorems

We define the following function space

$$X_T := \{u \in \mathcal{C}([-T,T]; \mathcal{S}') \,:\, \|u\|_{X_T} < \infty\} , \qquad (2.1)$$

where $\mathcal{S}'$ denotes the space of tempered distributions and

$$\begin{aligned} \|u\|_{X_T} = & \sup_{t\in[-T,T]} (1+|t|)^{-C(\varepsilon_1^2+\varepsilon_1^p)} (\|u(t)\|_{\gamma,0} + \|S(-t)u(t)\|_{0,\gamma}) \\ & + \sup_{t\in[-T,T]} (1+|t|)^{\frac{3}{2}} \|u(t)\|_{\infty} . \end{aligned} \qquad (2.2)$$

We first recall some estimates which will be used to prove our results.

Lemma 2.1. (Lemma 2.2[14]) *Let* $u(t,x)$ *be a smooth function. Then we have*

$$\begin{aligned} \|u(t)\|_{\infty} \le & C|t|^{-3/2} \|\mathcal{F}S(-t)u(t)\|_{\infty} \\ & + C|t|^{-3/2-\alpha} \|S(-t)u(t)\|_{0,\gamma}, \quad \textit{for } |t| \ge 1 , \end{aligned}$$

where $\alpha \in [0,1)$, $\gamma > 3/2 + 2\alpha$*.*

Proof. (cf. Lemma 2.2[14]) We have the identity

$$u(t,x) = S(t)S(-t)u(t,x) = (2\pi i t)^{-n/2} \int e^{i|x-y|^2/2t} S(-t)u(t,y)\,dy . \qquad (2.3)$$

The identity (2.3) can be written as follows:

$$\begin{aligned} u(t,x) &= \frac{e^{i|x|^2/2t}}{(2\pi i t)^{n/2}} \int e^{-ixy/t} S(-t)u(x,y) \{1 + (e^{i|y|^2/2t} - 1)\}\,dy \\ &= \frac{e^{i|x|^2/2t}}{(it)^{n/2}} (\mathcal{F}S(-t)u(t)) \left(t, \frac{x}{t}\right) + R(t,x) , \end{aligned} \qquad (2.4)$$

where

$$R(t,x) = \frac{e^{i|x|^2/2t}}{(2\pi i t)^{n/2}} \int e^{-ixy/t} (e^{i|y|^2/2t} - 1) S(-t)u(t,y)\,dy .$$

Let us state a basic inequality which will be used in our argument. Let $\alpha \in (0,1)$ be a fixed number, then we have

$$|M(-t) - 1| = \left| e^{i\frac{|x|^2}{2t}} - 1 \right| \le C \left(\frac{|x|^2}{2|t|} \right)^{\alpha} . \qquad (2.5)$$

In fact, it is clear that (2.5) holds for $\frac{|x|^2}{2|t|} \geq 2^{1/\alpha}$. Now, we assume that $\frac{|x|^2}{2|t|} < 2^{1/\alpha}$. Thus,

$$\begin{aligned} |M(-t)-1| &= \left|e^{i\frac{|x|^2}{2t}} - 1\right| = \left|\cos\frac{|x|^2}{2t} + i\sin\frac{|x|^2}{2t} - 1\right| = \left(2 - 2\cos\frac{|x|^2}{2t}\right)^{1/2} \\ &= 2\left|\sin\frac{|x|^2}{4t}\right| \leq 2\left|\frac{|x|^2}{4t}\right| \leq 2^{\frac{1-\alpha}{\alpha}}\left(\frac{|x|^2}{2|t|}\right)^{\alpha} \leq C\left(\frac{|x|^2}{2|t|}\right)^{\alpha}. \end{aligned} \tag{2.6}$$

Hence we have by the Schwartz inequality

$$\begin{aligned} \|R(t)\|_\infty &\leq C|t|^{-3/2-\alpha}\||y|^{2\alpha}S(-t)u(t,y)\|_1 \\ &\leq C|t|^{-3/2-\alpha}\|S(-t)u(t)\|_{0,\gamma}, \quad \text{for } |t| \geq 1, \end{aligned} \tag{2.7}$$

where $\gamma > 3/2 + 2\alpha$. From (2.4) and (2.7), the lemma follows. □

Lemma 2.2. (cf. Lemma 2.4[14]) *Let* $\gamma > 0$. *Then we have*

$$\begin{aligned} &|\mathrm{Im}(|x|^\gamma S(-t)(|x|^{-1} * |u|^2)v(t), |x|^\gamma S(-t)v(t))| \\ &\quad \leq C\|u\|_\infty^{1/3}\|u\|_2^{2/3}\|v\|_\infty^{1/3}\|v\|_2^{2/3}\||x|^\gamma S(-t)u\|_2\||x|^\gamma S(-t)v\|_2, \end{aligned}$$

where the $(\cdot,\cdot)$ *denotes the inner product in* L^2.

Proof. We prove the case $0 < \gamma < 1$. By the relation $M(t)(-t^2\Delta)^{\gamma/2}M(-t) = S(t)|x|^\gamma S(-t)$, the results about fractional derivatives in Ref. 18 and Young's inequality, we have, for $g = M(-t)v$ and the real function $f = |x|^{-1} * |u|^2$, that

$$\begin{aligned} &|\mathrm{Im}(|x|^\gamma S(-t)(|x|^{-1} * |u|^2)v(t), |x|^\gamma S(-t)v(t))| \\ &\quad = |\mathrm{Im}((-t^2\Delta)^{\gamma/2}(fg), (-t^2\Delta)^{\gamma/2}g)| \\ &\quad = |\mathrm{Im}((-t^2\Delta)^{\gamma/2}(fg) - f(-t^2\Delta)^{\gamma/2}g, (-t^2\Delta)^{\gamma/2}g)| \\ &\quad \leq C\|g\|_3\|(-t^2\Delta)^{\gamma/2}f\|_6\|(-t^2\Delta)^{\gamma/2}g\|_2 \\ &\quad \leq C\|g\|_3\||x|^{-1} * (-t^2\Delta)^{\gamma/2}|u|^2\|_6\|(-t^2\Delta)^{\gamma/2}g\|_2 \\ &\quad \leq C\|g\|_3\|(-t^2\Delta)^{\gamma/2}|u|^2\|_{6/5}\|(-t^2\Delta)^{\gamma/2}g\|_2 \\ &\quad \leq C\|g\|_3\|(-t^2\Delta)^{\gamma/2}M(-t)u\|_2\|u\|_3\|(-t^2\Delta)^{\gamma/2}g\|_2 \\ &\quad \leq C\|v\|_\infty^{1/3}\|v\|_2^{2/3}\||x|^\gamma S(-t)u\|_2\|u\|_\infty^{1/3}\|u\|_2^{2/3}\||x|^\gamma S(-t)v\|_2. \end{aligned}$$

In the same way as in the case $0 < \gamma < 1$, we have the first estimate for general γ. □

The proof of our theorems consists of short time existence theorem and the *a priori* estimates of local in time solutions. Since the local existence of solutions (ψ_1, ψ_2) of (1.1)–(1.2) with (1.3) can be done in the framework,[4,13,14] we just list it below and omit the proof.

Lemma 2.3. *Let* $\sum_{j=1,2}[\|\phi_j\|_{\gamma,0}+\|\phi_j\|_{0,\gamma}] =: \varepsilon_1 \le \varepsilon$, *where* ε *is sufficiently small and* $3/2 < \gamma \le 5/3$. *Then there exists a finite time interval* $[-T,T]$ *with* $T > 1$ *and a unique solution* (ψ_1,ψ_2) *of* (1.1)–(1.2) *with* (1.3) *such that*

$$\|\psi_j\|_{X_T} \le C\varepsilon\,,$$

where the constant C *is independent of* T *and* ε.

What is left for us is to establish the *a priori* estimates of the local solutions to the Schrödinger–Poisson equations (1.1)–(1.2). In fact, we can obtain the following *a priori* estimates on the local solutions.

Lemma 2.4. *Let* ψ_j *be the local solutions to* (1.1)–(1.2) *with* (1.3) *stated in Lemma* 2.3. *Then we have for any* $t \in [-T,T]$ *and* $3/2 < \gamma \le 5/3$

$$\begin{aligned}(1+|t|)^{-C(\varepsilon^2+\varepsilon^p)} &\sum_{j=1,2}(\|\psi_j(t)\|_{\gamma,0}+\|S(-t)\psi_j(t)\|_{0,\gamma})\\ &\le C\sum_{j=1,2}(\|\phi_j\|_{\gamma,0}+\|\phi_j\|_{0,\gamma}) \equiv C\varepsilon_1\,,\end{aligned}$$

where the constant C *is independent of* T, ε *and* ε_1.

Proof. Multiplying both sides of (1.1) by $|J|^\gamma = S(t)|x|^\gamma S(-t)$ and using the commutation relation $[L,|J|^\gamma]=0$ with $L = i\partial_t + \frac{1}{2}\Delta$, we obtain

$$L|J|^\gamma\psi_j = |J|^\gamma[q_jV(\psi_1,\psi_2)\psi_j + a_j^2|\psi_j|^p\psi_j]\,.$$

Multiplying both sides of the above equation by $\overline{|J|^\gamma\psi_j}$ and integrating the resulting equation, we have from Lemma 2.2

$$\begin{aligned}\||J|^\gamma\psi_j(t)\|_2^2 &\le \||x|^\gamma\phi_j\|_2^2 + C\,\mathrm{Im}\int_0^t\int_{\mathbb{R}^3}|J|^\gamma[q_jV(\psi_1,\psi_2)\psi_j + a_j^2|\psi_j|^p\psi_j]\overline{|J|^\gamma\psi_j}\,dx\,d\tau\\ &\le \||x|^\gamma\phi_j\|_2^2 + C\int_0^t\left(\varepsilon^{4/3}\sum_{j=1,2}\|\psi_j(\tau)\|_\infty^{2/3} + a_j^2\|\psi_j(\tau)\|_\infty^p\right)\||J|^\gamma\psi_j\|_2^2\,d\tau\\ &\le \||x|^\gamma\phi_j\|_2^2 + C(\varepsilon^2+\varepsilon^p)\int_0^t(1+\tau)^{-1}\||J|^\gamma\psi_j\|_2^2\,d\tau\,.\end{aligned}$$

Applying the Gronwall inequality, we get

$$\||J|^\gamma\psi_j\|_2^2 \le \||x|^\gamma\phi_j\|_2^2(1+t)^{C(\varepsilon^2+\varepsilon^p)}\,,$$

which implies

$$(1+t)^{-C(\varepsilon^2+\varepsilon^p)}\sum_{j=1,2}\|S(-t)\psi_j(t)\|_{0,\gamma} \le C\sum_{j=1,2}\|\phi_j\|_{0,\gamma} \le C\varepsilon_1\,.$$

In the same way as in the proof of the above, we have

$$(1+|t|)^{-C(\varepsilon^2+\varepsilon^p)}\sum_{j=1,2}\|\psi_j(t)\|_{\gamma,0} \le C\sum_{j=1,2}(\|\phi_j\|_{\gamma,0}+\|\phi_j\|_{0,\gamma}) \equiv C\varepsilon_1\,. \qquad \square$$

Lemma 2.5. *Let ψ_1, ψ_2 be the local solutions to* (1.1)–(1.2) *with* (1.3). *Then we have for $t \in [-T, T]$*

$$(1+|t|)^{\frac{3}{2}}\|\psi_j(t)\|_\infty \le C\sum_{j=1}^{2}(\|\phi_j\|_{\gamma,0} + \|\phi_j\|_{0,\gamma}) \equiv C\varepsilon_1\,, \tag{2.8}$$

where the constant C is independent of T and ε_1.

Proof. It is clear that (2.8) holds for $|t| \le 1$ due to the Sobolev embedding.[1] Let us turn to $|t| > 1$ next. By Lemmas 2.1 and 2.4, we can get

$$\begin{aligned}\|\psi_j(t)\|_\infty \le{}& C|t|^{-\frac{3}{2}-\alpha+C(\varepsilon_1^2+\varepsilon_1^p)}(\|\phi_j\|_{\gamma,0} + \|\phi_j\|_{0,\gamma})\\ &+ C|t|^{-\frac{3}{2}}\|\mathcal{F}S(-t)\psi_j(t)\|_\infty\,,\end{aligned} \tag{2.9}$$

where $0 < \alpha < 1$ and $\gamma > \frac{3}{2} + 2\alpha$. We now deal with the last term of the R.H.S. of (2.9). Multiplying both sides of (1.1) by $S(-t)$, we have

$$iS(-t)\dot{\psi}_j = -\frac{1}{2}S(-t)\Delta\psi_j + S(-t)(q_jV + a_j^2|\psi_j|^p)\psi_j\,. \tag{2.10}$$

Let $v_j = S(-t)\psi_j$. Noticing that $S(-t) = M(-t)\mathcal{F}^{-1}D(t)^{-1}M(-t)$ and

$$\begin{aligned}(f * g)(tx) &= \int_{\mathbb{R}^3} f(tx-y)g(y)\,dy\\ &= \int_{\mathbb{R}^3} f(tx-tz)g(tz)t^3\,dz\\ &= t^3 f(tx) * g(tx)\,,\end{aligned} \tag{2.11}$$

we have the identity

$$\begin{aligned}S(-t)V\psi_j &= M(-t)\mathfrak{F}^{-1}D(t)^{-1}M(-t)V\psi_j\\ &= M(-t)\mathfrak{F}^{-1}D(t)^{-1}V(M(-t)\psi_1, M(-t)\psi_2)M(-t)\psi_j\\ &= M(-t)\mathfrak{F}^{-1}i^3 D\left(\frac{1}{t}\right)V(M(-t)\psi_1, M(-t)\psi_2)M(-t)\psi_j\\ &= M(-t)\mathfrak{F}^{-1}i^3(it^{-1})^{-\frac{3}{2}}\left[\frac{Ct^3}{|tx|} * (|(M(-t)\psi_1)(tx)|^2\right.\\ &\quad \left. -|(M(-t)\psi_2)(tx)|^2)\right](M(-t)\psi_j)(tx)\\ &= M(-t)\mathfrak{F}^{-1}t^{-1}\left[\frac{C}{|x|} * (|D(t)^{-1}M(-t)\psi_1|^2\right.\\ &\quad \left. -|D(t)^{-1}M(-t)\psi_2|^2)\right]D(t)^{-1}M(-t)\psi_j\\ &= t^{-1}M(-t)\mathfrak{F}^{-1}[V(D(t)^{-1}M(-t)\psi_1, D(t)^{-1}M(-t)\psi_2)D(t)^{-1}M(-t)\psi_j]\\ &= t^{-1}M(-t)\mathfrak{F}^{-1}[V(\widehat{M(t)v_1}, \widehat{M(t)v_2})\widehat{M(t)v_j}]\end{aligned}$$

$$= t^{-1}\big\{(M(-t)-1)\mathcal{F}^{-1}[V(\widehat{M(t)v_1}, \widehat{M(t)v_2})\widehat{M(t)v_j}] + \mathcal{F}^{-1}[V(\widehat{M(t)v_1}, \widehat{M(t)v_2})\widehat{M(t)v_j} - V(\widehat{v}_1, \widehat{v}_2)\widehat{v}_j]\big\} + t^{-1}\mathcal{F}^{-1}V(\widehat{v}_1, \widehat{v}_2)\widehat{v}_j\,. \tag{2.12}$$

Thus, we obtain

$$i\dot{v}_j - q_j t^{-1}\mathcal{F}^{-1}[V(\widehat{v}_1, \widehat{v}_2)\widehat{v}_j] = q_j\mathcal{F}^{-1}t^{-1}(I_1 + I_2) + Q_j$$

and

$$i\dot{\hat{v}}_j - q_j t^{-1}[V(\widehat{v}_1, \widehat{v}_2)\widehat{v}_j] = q_j t^{-1}(I_1 + I_2) + \hat{Q}_j\,,$$

where

$$\begin{aligned} I_1 &= \mathcal{F}(M(-t)-1)\mathcal{F}^{-1}V(\widehat{M(t)v_1}, \widehat{M(t)v_2})\widehat{M(t)v_j}\,,\\ I_2 &= V(\widehat{M(t)v_1}, \widehat{M(t)v_2})\widehat{M(t)v_j} - V(\widehat{v}_1, \widehat{v}_2)\widehat{v}_j\,,\\ Q_j &= a_j^2 S(-t)|\psi_j|^p\psi_j\,. \end{aligned}$$

Let $B_j(t) = \exp(iq_j\int_1^t V(\hat{v}_1, \hat{v}_2)\frac{d\tau}{\tau})$ and $\hat{w}_j = B_j\hat{v}_j$. Then, we have

$$i\dot{\hat{w}}_j = B_j(t)[q_j t^{-1}(I_1 + I_2) + \hat{Q}_j(t)]\,.$$

Integrating over [1,t] with respect to the time variable t, we can get

$$\hat{w}_j(t) = \hat{w}_j(1) - i\int_1^t B_j(t)[q_j\tau^{-1}(I_1 + I_2) + \hat{Q}_j(\tau)]\,d\tau\,. \tag{2.13}$$

Let $h_j = M(t)v_j$, then, by the Hausdorff–Young inequality, we have

$$\begin{aligned} \|I_1\|_\infty &= \|\mathcal{F}(M(-t)-1)\mathcal{F}^{-1}V(\hat{h}_1, \hat{h}_2)\hat{h}_j\|_\infty\,,\\ &\le \|(M(-t)-1)\mathcal{F}^{-1}V(\hat{h}_1, \hat{h}_2)\hat{h}_j\|_1\,. \end{aligned}$$

Using the inequality (2.5), it can be shown for $\gamma' > 3/2$ and $\varepsilon' \in (0, 2\alpha]$, that

$$\begin{aligned} \|I_1\|_\infty &\le |t|^{-\alpha}\||x|^{2\alpha}\mathcal{F}^{-1}V(\hat{h}_1, \hat{h}_2)\hat{h}_j\|_1 \le C|t|^{-\alpha}\|V(\hat{h}_1, \hat{h}_2)\hat{h}_j\|_{\gamma,0}\\ &\le C|t|^{-\alpha}\|[(-\Delta)^{-1}(|\hat{h}_1|^2 - |\hat{h}_2|^2)]\hat{h}_j\|_{\gamma,0}\\ &\le C|t|^{-\alpha}(\|\hat{h}_1\|^2_{\gamma'+\varepsilon',0} + \|\hat{h}_2\|^2_{\gamma'+\varepsilon',0})\|\hat{h}_j\|_{\gamma,0}\\ &\le C|t|^{-\alpha}(\|h_1\|^2_{0,\gamma'+\varepsilon'} + \|h_2\|^2_{0,\gamma'+\varepsilon'})\|h_j\|_{0,\gamma}\\ &\le C|t|^{-\alpha}(\|v_1\|^2_{0,\gamma'+\varepsilon'} + \|v_2\|^2_{0,\gamma'+\varepsilon'})\|v_j\|_{0,\gamma} \end{aligned} \tag{2.14}$$

and

$$\begin{aligned} \|I_2\|_\infty &= \|V(\hat{h}_1, \hat{h}_2)\hat{h}_j - V(\hat{v}_1, \hat{v}_2)\hat{v}_j\|_\infty\\ &= C\left\|\left[|x|^{-1} * (|\hat{h}_1|^2 - |\hat{h}_2|^2)\right]\hat{h}_j - \left[|x|^{-1} * (|\hat{v}_1|^2 - |\hat{v}_2|^2)\right]\hat{v}_j\right\|_\infty\\ &\le C\left\{\left\|(|x|^{-1} * |\hat{h}_1|^2)\hat{h}_j - (|x|^{-1} * |\hat{v}_1|^2)\hat{v}_j\right\|_\infty\right. \end{aligned}$$

$$
\begin{aligned}
&\quad + \left\| (|x|^{-1} * |\hat{h}_2|^2)\hat{h}_j - (|x|^{-1} * |\hat{v}_2|^2)\hat{v}_j \right\|_\infty \Big\} \\
&\le C \left\{ \left\| (|x|^{-1} * |\hat{h}_1|^2)(\hat{h}_j - \hat{v}_j) + (|x|^{-1} * (|\hat{h}_1|^2 - |\hat{v}_1|^2))\hat{v}_j \right\|_\infty \right. \\
&\quad \left. + \|(|x|^{-1} * |\hat{h}_2|^2)(\hat{h}_j - \hat{v}_j) + (|x|^{-1} * (|\hat{h}_2|^2 - |\hat{v}_2|^2))\hat{v}_j\|_\infty \right\} \\
&\le C \Big\{ \||x|^{-1} * |\hat{h}_1|^2\|_\infty \|\hat{h}_j - \hat{v}_j\|_\infty + \|(|x|^{-1} * (\hat{h}_1 - \hat{v}_1))\bar{\hat{h}}_1\|_\infty \|\hat{v}_j\|_\infty \\
&\quad + \|(|x|^{-1} * (\bar{\hat{h}}_1 - \bar{\hat{v}}_1))\hat{v}_1\|_\infty \|\hat{v}_j\|_\infty + \||x|^{-1} * |\hat{h}_2|^2\|_\infty \|\hat{h}_j - \hat{v}_j\|_\infty \\
&\quad + \|(|x|^{-1} * (\hat{h}_2 - \hat{v}_2))\bar{\hat{h}}_2\|_\infty \|\hat{v}_j\|_\infty + \|(|x|^{-1} * (\bar{\hat{h}}_2 - \bar{\hat{v}}_2))\hat{v}_2\|_\infty \|\hat{v}_j\|_\infty \Big\} .
\end{aligned}
$$

To continue the above estimate, we introduce the following lemma.

Lemma 2.6. *Let $u \in L^q(\mathbb{R}^3) \cap L^r(\mathbb{R}^3)$ for $1 < r < 3 < q < \infty$ and $1/q + 1/r = 1$. Then, we have*

$$
\||x|^{-1} * u\|_\infty \le C(\|u\|_q + \|u\|_r) .
$$

Proof. By Hölder's inequality, we have

$$
\begin{aligned}
\||x|^{-1} * u\|_\infty &= \sup_x \left| \int |y|^{-1} u(x-y)\, dy \right| \\
&\le \sup_x \left| \int_{|y|<1} |y|^{-1} u(x-y)\, dy \right| + \sup_x \left| \int_{|y|\ge 1} |y|^{-1} u(x-y)\, dy \right| \\
&\le \left(\int_{|y|<1} |y|^{-r}\, dy \right)^{1/r} \|u\|_q + \left(\int_{|y|\ge 1} |y|^{-q}\, dy \right)^{1/q} \|u\|_r \\
&\le C(\|u\|_q + \|u\|_r) .
\end{aligned}
$$

□

With the help of Lemma 2.6, we obtain

$$
\begin{aligned}
\|I_2\|_\infty &\le C\{((\|\hat{h}_1\|_{2q} + \|\hat{v}_1\|_{2q})\|\hat{h}_1 - \hat{v}_1\|_{2q} + (\|\hat{h}_1\|_{2r} + \|\hat{v}_1\|_{2r})\|\hat{h}_1 - \hat{v}_1\|_{2r})\|\hat{v}_j\|_\infty \\
&\quad + (\|\hat{h}_1\|_{2q}^2 + \|\hat{h}_1\|_{2r}^2)\|\hat{h}_j - \hat{v}_j\|_\infty + (\|\hat{h}_2\|_{2q}^2 + \|\hat{h}_2\|_{2r}^2)\|\hat{h}_j - \hat{v}_j\|_\infty \\
&\quad + ((\|\hat{h}_2\|_{2q} + \|\hat{v}_2\|_{2q})\|\hat{h}_2 - \hat{v}_2\|_{2q} + (\|\hat{h}_2\|_{2r} + \|\hat{v}_2\|_{2r})\|\hat{h}_2 - \hat{v}_2\|_{2r})\|\hat{v}_j\|_\infty\} \\
&\le C\{(\|\hat{h}_1\|_{3/2,0} + \|\hat{v}_1\|_{3/2,0})\|\mathcal{F}(M(t)-1)v_1\|_{3/2,0}\|\hat{v}_j\|_\infty \\
&\quad + (\|\hat{h}_1\|_{3/2,0}^2 + \|\hat{h}_2\|_{3/2,0}^2)\|(M(t)-1)v_j\|_{0,1} \\
&\quad + (\|\hat{h}_2\|_{3/2,0} + \|\hat{v}_2\|_{3/2,0})\|\mathcal{F}(M(t)-1)v_2\|_{3/2,0}\|\hat{v}_j\|_\infty\} \\
&\le C|t|^{-\alpha} \sum_{j=1,2} \|v_j\|_{0,\gamma}^3 .
\end{aligned} \tag{2.15}
$$

Analogous to the estimate (2.14), we also have for $\gamma' > 3/2$

$$\begin{aligned}\|\hat{Q}_j(t)\|_\infty &= a_j^2|t|^{-\frac{3p}{2}}\|\mathcal{F}M(-t)\mathcal{F}^{-1}|\hat{h}_j|^p\hat{h}_j\|_\infty \\ &\le C|t|^{-\frac{3p}{2}}\|\mathcal{F}^{-1}|\hat{h}_j|^p\hat{h}_j\|_1 \le C|t|^{-\frac{3p}{2}}\||\hat{h}_j|^p\hat{h}_j\|_{\gamma',0} \\ &\le C|t|^{-\frac{3p}{2}}\|\hat{h}_j\|_\infty^p\|\hat{h}_j\|_{\gamma',0} \le C|t|^{-\frac{3p}{2}}\|h_j\|_1^p\|h_j\|_{0,\gamma'} \\ &\le C|t|^{-\frac{3p}{2}}\|h_j\|_{0,\gamma'}^{p+1}\,. \end{aligned} \tag{2.16}$$

By (2.14)–(2.16), we can get

$$\begin{aligned}\|\mathcal{F}S(-t)\psi_j(t)\|_\infty &= \|\hat{v}_j\|_\infty = \|B_j^{-1}\hat{w}_j\|_\infty \\ &\le \|\hat{w}_j(1)\|_\infty + \int_1^t \tau^{-1}(\|I_1\|_\infty + \|I_2\|_\infty) + \|\hat{Q}_j\|_\infty\, d\tau \\ &\le \|\hat{w}_j(1)\|_\infty + C\int_1^t |\tau|^{-1}|\tau|^{-\alpha}((1+|\tau|)^{C(\varepsilon^2+\varepsilon^p)}\varepsilon_1)^3 \\ &\quad + |\tau|^{-\frac{3p}{2}}((1+|\tau|)^{C(\varepsilon^2+\varepsilon^p)}\varepsilon_1)^{p+1}\, d\tau \\ &\le \|\hat{w}_j(1)\|_\infty + C\varepsilon_1\int_1^t |\tau|^{-1-\alpha+C(\varepsilon^2+\varepsilon^p)} + |\tau|^{-\frac{3p}{2}+C(\varepsilon^2+\varepsilon^p)}\, d\tau \\ &\le \|\hat{w}_j(1)\|_\infty + C\varepsilon_1\,, \end{aligned}$$

and

$$\begin{aligned}\|\hat{w}_j(1)\|_\infty &= \|\hat{v}_j(1)\|_\infty = \|\mathcal{F}S(-1)\psi_j(1)\|_\infty \\ &\le \|\mathcal{F}S(-1)\psi_j(1)\|_{\gamma,0} = \|S(-1)\psi_j(1)\|_{0,\gamma}\,. \end{aligned}$$

Thus, we obtain

$$\|\mathcal{F}S(-t)\psi_j(t)\|_\infty \le C\varepsilon_1\,.$$

Therefore, we have the following estimate

$$\|\psi_j\|_\infty \le C\varepsilon_1|t|^{-\frac{3}{2}}$$

which implies the desired result (2.8). □

Similar to Lemma 2.4, we can obtain the following lemma in view of Lemma 2.5.

Lemma 2.7. *Let ψ_j be the local solutions to* (1.1)–(1.2) *with* (1.3) *stated in Lemma* 2.3. *Then we have for any $t \in [-T, T]$ and $3/2 < \gamma \le 5/3$*

$$\begin{aligned}(1+|t|)^{-C(\varepsilon_1^2+\varepsilon_1^p)} &\sum_{j=1,2}(\|\psi_j(t)\|_{\gamma,0} + \|S(-t)\psi_j(t)\|_{0,\gamma}) \\ &\le C\sum_{j=1,2}(\|\phi_j\|_{\gamma,0} + \|\phi_j\|_{0,\gamma}) \equiv C\varepsilon_1\,, \end{aligned}$$

where the constant C is independent of T and ε_1.

Proof of Theorems 1.1–1.2. We now prove our main results — Theorems 1.1 and 1.2. Following the ideas of Hayashi *et al*,[13,14] with the help of Lemmas 2.5 and 2.7, we can easily obtain the results in Theorem 1.1. We omit the details.

We prove Theorem 1.2 next. We only consider the case $t > 0$ since the opposite case can be treated analogously. From (2.13), (2.14), (2.15) and (2.16), we have

$$\begin{aligned} |\hat{w}_j(t) - \hat{w}_j(s)| &\le C \int_s^t [\tau^{-1}(\|I_1(\tau)\|_\infty + \|I_2(\tau)\|_\infty) + \|\hat{Q}_j(\tau)\|_\infty]\, d\tau \\ &\le C\varepsilon_1 \int_s^t [\tau^{-1-\alpha+C(\varepsilon_1^2+\varepsilon_1^p)} + \tau^{-\frac{3p}{2}+C(\varepsilon_1^2+\varepsilon_1^p)}]\, d\tau \\ &\le C\varepsilon_1 (s^{-\alpha+C(\varepsilon_1^2+\varepsilon_1^p)} - t^{-\alpha+C(\varepsilon_1^2+\varepsilon_1^p)} \\ &\quad + s^{1-\frac{3p}{2}+C(\varepsilon_1^2+\varepsilon_1^p)} - t^{1-\frac{3p}{2}+C(\varepsilon_1^2+\varepsilon_1^p)}) \end{aligned} \tag{2.17}$$

and we find that $\{\hat{w}_j(t)\}$ is such a Cauchy sequence that there exist a unique pair of functions $(\mathcal{W}_1, \mathcal{W}_2)^+$ with $\mathcal{W}_1^+$, $\mathcal{W}_2^+ \in L^\infty$ such that

$$\|\mathcal{W}_j^+ - \hat{w}_j(t)\|_\infty \le C\varepsilon_1 (t^{-\alpha+C(\varepsilon_1^2+\varepsilon_1^p)} + t^{1-\frac{3p}{2}+C(\varepsilon_1^2+\varepsilon_1^p)}), \quad \text{for } j = 1, 2\,.$$

Let

$$\Gamma^+(t) = \int_1^t [V(\hat{w}_1(\tau), \hat{w}_2(\tau)) - V(\hat{w}_1(t), \hat{w}_2(t))] \frac{d\tau}{\tau}\,.$$

Then, we have

$$\begin{aligned} \Gamma^+(t) - \Gamma^+(s) &= \int_s^t [V(\hat{w}_1(\tau), \hat{w}_2(\tau)) - V(\hat{w}_1(t), \hat{w}_2(t))] \frac{d\tau}{\tau} \\ &\quad + [V(\hat{w}_1(t), \hat{w}_2(t)) - V(\hat{w}_1(s), \hat{w}_2(s))] \ln s\,. \end{aligned} \tag{2.18}$$

By the Hölder inequality, we have for $1 < r < 3 < q$

$$\begin{aligned} &\|V(\hat{w}_1(t), \hat{w}_2(t)) - V(\hat{w}_1(s), \hat{w}_2(s))\|_\infty \\ &\quad \le C[\||\hat{w}_1(t)|^2 - |\hat{w}_2(t)|^2 - |\hat{w}_1(s)|^2 + |\hat{w}_2(s)|^2\|_q \\ &\qquad + \||\hat{w}_1(t)|^2 - |\hat{w}_2(t)|^2 - |\hat{w}_1(s)|^2 + |\hat{w}_2(s)|^2\|_r] \\ &\quad \le C \sum_{j=1,2} [\|\hat{w}_j(t) - \hat{w}_j(s)\|_\infty^{2-2/q} (\|\hat{w}_j(t)\|^{2/q} + \|\hat{w}_j(s)\|^{2/q}) \\ &\qquad + \|\hat{w}_j(t) - \hat{w}_j(s)\|_\infty (\|\hat{w}_j(t)\|_\infty^{1-2/r} \\ &\qquad + \|\hat{w}_j(s)\|_\infty^{1-2/r})(\|\hat{w}_j(t)\|_\infty^{2/r} + \|\hat{w}_j(s)\|_\infty^{2/r})]\,. \end{aligned}$$

With (2.17), the above yields

$$\begin{aligned} \|V(\hat{w}_1(t), \hat{w}_2(t)) - V(\hat{w}_1(s), \hat{w}_2(s))\|_\infty &\le C\varepsilon_1 (s^{-\alpha+C(\varepsilon_1^2+\varepsilon_1^p)} - t^{-\alpha+C(\varepsilon_1^2+\varepsilon_1^p)} \\ &\quad + s^{1-\frac{3p}{2}+C(\varepsilon_1^2+\varepsilon_1^p)} - t^{1-\frac{3p}{2}+C(\varepsilon_1^2+\varepsilon_1^p)})^\theta\,, \end{aligned} \tag{2.19}$$

for certain constant $0 < \theta < 2/3$.

Hence, by (2.18), we see that there exists a unique function $\Lambda^+ \in L^\infty$ satisfying

$$\|\Lambda^+ - \Gamma^+(t)\|_\infty \le C\varepsilon_1 (t^{-\alpha + C(\varepsilon_1^2 + \varepsilon_1^p)} + t^{1 - \frac{3p}{2} + C(\varepsilon_1^2 + \varepsilon_1^p)})^\theta. \tag{2.20}$$

By (2.20) and the identity

$$\begin{aligned}\int_1^t q_j V(\hat{w}_1(\tau), \hat{w}_2(\tau)) \frac{d\tau}{\tau} &= q_j V(\mathcal{W}_1^+, \mathcal{W}_2^+) \ln t + \Lambda^+ + (\Gamma^+(t) - \Lambda^+) \\ &\quad + q_j (V(\hat{w}_1(\tau), \hat{w}_2(\tau)) - V(\mathcal{W}_1^+, \mathcal{W}_2^+)) \ln t\,, \end{aligned} \tag{2.21}$$

we obtain the desired result (1.10) from the restriction stated for α and p. By (1.9) and (1.10), we have (1.11). Thus, we have completed the proof of Theorem 1.2. □

Acknowledgment

The authors would like to thank the referees for valuable comments and suggestions on an early manuscript of this paper. The second author was partially supported by the MST (Grant No. 1999075107), NSAF (Grant No. 10276036) and the Innovation funds of AMSS, CAS of China.

References

1. J. Bergh and J. Löfström, *Interpolation Spaces, An Introduction* (Springer-Verlag, 1976).
2. J. Bourgain, Global wellposedness of defocusing critical nonlinear Schrödinger equation in the radial case, *J. Amer. Math. Soc.* **12** (1999) 145–171.
3. F. Brezzi and P. A. Markowich, The three-dimensional Wigner–Poisson problem: Existence, uniqueness and approximation, *Math. Meth. Appl. Sci.* **14** (1991) 35–62.
4. F. Castella, L^2 solutions to the Schrödinger–Poisson system: Existence, uniqueness, time behaviour, and smoothing effects, *Math. Mod. Meth. Appl. Sci.* **7** (1997) 1051–1083.
5. J. Colliander, M. Keel, G. Staffilani, H. Takaoka and T. Tao, Global well-posedness and scattering for the energy-critical nonlinear Schrödinger equation in $\mathbb{R}^3$, preprint, 2004.
6. J. Dias and M. Figueira, Conservation laws and time decay for the solutions of some nonlinear Schrödinger–Hartree equations and systems, *J. Math. Anal. Appl.* **84** (1981) 486–508.
7. J. Ginibre and T. Ozawa, Long range scattering for nolinear Schrödinger and Hartree equations in space dimension $n \ge 2$, *Comm. Math. Phys.* **151** (1993) 619–645.
8. J. Ginibre and G. Velo, Long range scattering and modified wave operators for some Hartree type equations I, *Rev. Math. Phys.* **12** (2000) 361–429.
9. J. Ginibre and G. Velo, Long range scattering and modified wave operators for some Hartree type equations II, *Ann. Inst. H. Poincaré* **1** (2000) 753–800.
10. J. Ginibre and G. Velo, Long range scattering and modified wave operators for some Hartree type equations III, Gevrey spaces and low dimensions, *J. Diff. Eqns.* **175** (2001) 415–501.
11. J. Ginibre and G. Velo, On a class of nonlinear Schrödinger equations, *J. Funct. Anal.* **32** (1971) 1–71.
12. J. Ginibre and G. Velo, Scattering theory in the energy space for a class of nonlinear Schrödinger equations, *J. Math. Pures Appl.* **64** (1985) 363–401.

13. C. C. Hao and L. Hsiao, Large-time behavior and global existence of solution to the bipolar defocusing nonlinear Schrödinger–Poisson system, accepted for publication in *Quart. Appl. Math.* (2003).
14. N. Hayashi and P. I. Naumkin, Asymptotics for large time of solutions to the nonlinear Schrödinger and Hartree equations, *Amer. J. Math.* **120** (1998) 369–389.
15. N. Hayashi and T. Ozawa, Time decay of solutions to the Cauchy problem for time-dependent Schrödinger–Hartree equations, *Comm. Math. Phys.* **110** (1987) 467–478.
16. J. Lin and W. Strauss, Decay and scattering of solutions of a nonlinear Schrödinger equation, *J. Funct. Anal.* **30** (1978) 245–263.
17. A. Jüngel and S. Wang, Convergence of nonlinear Schrödinger–Poisson systems to the compressible Euler equations, *Comm. Part. Diff. Eqns.* **28** (2003) 1005–1022.
18. C. E. Kenig, G. Ponce and L. Vega, Well-posedness and scattering results for the generalized Korteweg-de Vries equation via contraction principle, *Comm. Pure Appl. Math.* **46** (1993) 527–620.
19. H. L. Li and C. K. Lin, Semiclassical limit and well-posedness of nonlinear Schrödinger–Poisson, *Electronic J. Diff. Eqns.* **93** (2003) 1–17.
20. J. L. López and J. Soler, Asymptotic behavior to the 3-D Schrödinger/Hartree–Poisson and Wigner–Poisson systems, *Math. Mod. Meth. Appl. Sci.* **10** (2000) 923–943.
21. P. A. Markowich, G. Rein and G. Wolansky, Existence and nonlinear stability of stationary states of the Schrödinger–Poisson system, *J. Statist. Phys.* **106** (2002) 1221–1239.
22. P. A. Markowich, C. A. Ringhofer and C. Schmeiser, *Semiconductor Equations* (Springer-Verlag, 1990).
23. K. Nakanishi, Energy scattering for nonlinear Klein-Gordon and Schrödinger equations in spatial dimensions 1 and 2, *J. Funct. Anal.* **169** (1999) 201–225.
24. K. Nakanishi, Modified wave operators for the Hartree equation with data, image and convergence in the same space, *Comm. Pure. Appl. Anal.* **1** (2002) 237–252.
25. Y. Tsutsumi, Scattering problem for nonlinear Schrödinger equations, *Ann. Inst. H. Poincaré Phys. Theor.* **43** (1985) 321–347.
26. Y. Tsutsumi and K. Yajima, The asymptotic behavior of nonlinear Schrödinger equations, *Bull. Amer. Math. Soc.* **11** (1984) 186–188.